Multi-Mesh CFD

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Automated mesh adaptation to reduce numerical solution error has been a goal in CFD since the 1980s

- What is the proper quantity to drive mesh adaptation?
  - Driving by solution features (e.g., solution gradients, curvature) has been shown to fail “disastrously”
  - Examination of error transport equations shows truncation error (TE) is the local source of solution (discretization) error

- Example: 2D Burgers’ equation

**Governing PDE:**\[ u \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) - \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = 0 \]

**Error Transport Equation (simple linearization):**\[ u \left( \frac{\partial \varepsilon}{\partial x} + \frac{\partial \varepsilon}{\partial y} \right) - \nu \left( \frac{\partial^2 \varepsilon}{\partial x^2} + \frac{\partial^2 \varepsilon}{\partial y^2} \right) = \tau_h(u) \]
Prior work (AFOSR) shows that we can get good estimates of TE and solution error

- 2D Euler eqns (subsonic)
- Used a manufactured solution, so exact solution is known
- Solution error in density is much better than Richardson extrapolation
Two truncation error (TE) based approaches being used

- **Truncation error equidistribution**
  - Reduces TE by moving nodes to regions where it is high
  - Inexpensive, but limited effectiveness

- **Truncation error optimization**
  - Reduces TE by optimizing mesh node locations to reduce the L2 norm of TE
  - Expensive, but extremely effective for scalar equations...
Mesh Optimization†

TE optimization seeks to optimally place mesh nodes so that the TE is minimized

• Allows TE modification by changing both mesh resolution and mesh quality
• Can take advantage of cancellation of different TE terms
• Initially used mesh node locations as “design variables”
• Later switched to spring system with nodal forces “design variables” for more implicit adaptation behavior

† Joint work with Ed Alyanak, AFRL
2D Burgers’ equation

\[ u \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) - v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = 0 \]

Optimize mesh to minimize TE:

\[ J = \sqrt{\sum_i \sum_j (TE_{i,j})^2} \]

8x8 Cells: Re = 8

32x32 Cells: Re = 16
1D Euler equations solved for flow in a quasi-1D nozzle

- Governing equations involve coupled system for conservation of mass, x-momentum, and total energy
- Two nozzle geometries examined
  - Sine nozzle w/ straight sections appended (curvature discont.)
  - Gaussian nozzle
Mesh Adaptation Results*:
TE Equidistribution

Quasi-1D Euler eqns:

Sine Nozzle

Mesh Adaptation Results*:
TE Equidistribution

*Choudhary and Roy, 2013
(AIAA Paper 2013-2444)
Limitation: Each Governing Equation Desires a Different Mesh

Combined TE/Residual-Based Adaptation
### Mesh Optimization

**Quasi-1D nozzle: combining TE from three equations**

\[
J = \sqrt{\sum_i \left( \left( \frac{TE_{i,cont}}{TE_{max,cont}} \right)^2 + \left( \frac{TE_{i,xmtm}}{TE_{max,xmtm}} \right)^2 + \left( \frac{TE_{i,energy}}{TE_{max,energy}} \right)^2 \right)}
\]

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<tr>
<th>33 Nodes</th>
<th>Peak Continuity TE</th>
<th>Peak Momentum TE</th>
<th>Peak Energy TE</th>
<th>Improvement Factor (TE1)</th>
<th>Improvement Factor (TE2)</th>
<th>Improvement Factor (TE3)</th>
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Limitation: Each Governing Equation Desires a Different Mesh

Continuity

Energy

Combined TE Optimization
In multi-mesh CFD, we allow each governing equation to seek out its own optimal mesh

- Start with common initial grid
- Perform initial combined adaptation via TE equidistribution
- Begin multi-mesh optimization process
  - Adapt each mesh according to its own TE (via equidistribution or mesh optimization)
  - Solve continuity for $\rho$, x-mtm for $\rho u$, etc.
  - Must be able to reconstruct conserved variables onto other meshes (currently FDM, later FVM-consistent reconstruction)
  - Each conserved variable stored on each mesh
- Goal is to realize orders of magnitude reduction in discretization error
Finite difference code for solving the Euler equations in the quasi-1D nozzle using Jameson damping

- Second-order accurate finite differences
- Third-order interpolation across the meshes
- Currently using the exact truncation error
- Results are preliminary only
Adapted Grid: 129 nodes

Combined Weight Function

Individual Weight Function
Adaption Convergence – 129 nodes

Combined Weight Function

Individual Weight Function
TE on Adapted Grids

Combined Weight Function

Individual Weight Function
TE Comparison
Unstructured Grid Implications

Inviscid flow over an airfoil (2D Euler equations)

Exact truncation error for the mass conservation equation

Smooth Mesh  5% Perturbed  40% perturbed
Estimating Uncertainty in CFD Predictions

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There are three sources of uncertainty in CFD:

- **Uncertainty in model inputs**
  - Geometry
  - Boundary conditions
  - Fluid properties

- **Numerical errors (estimation → uncertainty)**
  - Discretization errors from grid and time step
  - Iterative errors
  - Statistical sampling errors

- **Modeling errors (estimation → uncertainty)**
  - “All models are wrong, some are useful” – George Box
  - Experimental measurement uncertainty
Aleatory (A) uncertainty – inherent variation in a quantity
- Characterized by PDF/CDF
- Propagate thru model by:
  - Sampling
  - Polynomial Chaos
  - Stochastic Collocation

Epistemic (E) uncertainty – due to a lack of knowledge
- Usually characterized by intervals: $X \in [A, B]$
- Propagate thru model by:
  - Sampling
  - Optimization

Combined A and E uncertainty
- Can be characterized w/ probability boxes
- Requires special treatment for propagation through the model
Discretization (i.e., grid-related) errors are still a large contributor to the overall uncertainty in CFD predictions

- Predictions are highly sensitive to grid topology/quality
- Error estimation based on Richardson extrap. is expensive
  - Requires three systematically-refined grids
  - All three meshes must be asymptotic to provide good estimates

Residual-based error estimation methods require two solutions on the same grid (AFOSR project)

- Local residual-based error estimation: error transport equations, defect correction (requires truncation error estim.)
- Adjoint methods for solution functionals: adjoint solution provides sensitivity of functional to local truncation errors
  - Estimation of numerical error results in epistemic uncertainty
Modeling uncertainty can be estimated using the Area Validation Metric (AVM):

\[ d(F, S_n) = \int_{-\infty}^{\infty} |F(Y) - S_n(Y)| dY \]

Model form uncertainty is epistemic and usually given in interval about simulation result \( F(Y) \): \([F(Y) - d, F(Y) + d]\)

- Improved version: Modified Area Validation Metric (MAVM)*

*Voyles and Roy, AIAA Paper 2014-0120
Validation Metric Comparison: Compressible Flow over an Airfoil

\[ \alpha = 4.0^\circ, M_\infty = 0.6; 1000 \text{ experimental samples; Random error only} \]
Validation Metric Comparison:
Compressible Flow over an Airfoil

Average Conservativeness over the Domain

Lift Coefficient

Drag Coefficient

Random error in inputs: $\sigma_\alpha = 0.5^\circ$, $\sigma_{M_\infty} = 0.002$; No bias error

*Voyles and Roy, AIAA Paper 2014-0120
Uncertainty increases when one accounts for:

- Probabilistic uncertainty (inputs)
- Epistemic/interval uncertainty (inputs)
- Modeling uncertainty
- Numerical uncertainty

Roy and Oberkampf (2011)
Thanks!